

Fundamentals of Mathematics

$$[x] = 5$$

$$x \in [5, 6)$$

$$[x] = -7$$

$$x \in [-7, -6)$$

$$\text{Q } [\log_3 x] = 5 \text{ then } x \in ?$$

$$\log_3 x \in [5, 6)$$

$$5 \leq \log_3 x < 6$$

$$3^5 \leq x < 3^6$$

$$x \in [3^5, 3^6)$$

$$\text{Q No of Integer from}$$

$$[7, 12) ?$$

$$= 12 - 7 \text{ Int.}$$

$$= 5 \text{ Int}$$

$$7, 8, 9, 10, 11$$

Fundamentals of Mathematics

Q (81, 243) No of Int?

$$243 - 81 = 162 \text{ Int.}$$

$$\begin{array}{r} 4771 \\ 5 \\ \hline 23855 \end{array}$$

Use of characteristic

→ Mantissa is basically

to find No. of digits

before Decimal &

No of zeros After Decimal.

Q No of Digits in 2^{100} ?

take $\log_{10}$

$$\log_{10} 2^{100}$$

$$= 100 \times \log_{10} 2$$

$$= 100 \times .3010$$

$$= 30.10 \rightarrow \begin{array}{l} \text{Ch} = 30 \\ \text{Mant} = .10 \end{array}$$

$$\begin{aligned} \text{No of Digits} &= (h+1) \\ &= (30+1) = 31 \\ &= 31 \end{aligned}$$

Q No of digits in 3^{50} ?

$$\log_{10} 3^{50}$$

$$50 \log_{10} 3$$

$$50 \times .4771$$

$$= 23.855 \rightarrow \begin{array}{l} \text{Ch} = 23 \\ \text{Mant} = .855 \end{array}$$

$$\begin{aligned} \text{No of Digits} &= (23+1) \\ &= 24 \\ &= 24 \end{aligned}$$

Fundamentals of Mathematics

Q No of digits in 6^{100} ?

$$\log_{10} 6^{100}$$

$$100 \log_{10} 6$$

$$100 \times \log_{10} (2 \times 3)$$

$$100 \times (\log_{10} 2 + \log_{10} 3)$$

$$100 \times (.3010 + .4771)$$

$$100 \times (.7781) \rightarrow Ch = 77$$

$$= 77.81 \rightarrow Mant = .81$$

$$\text{No of digits} = 77 + 1 = 78$$

Fractional Part fn.

(1) It is Rep. by $f(x) = \{x\}$

(2) It gives fractional value of No.

$$\{6.22\} = .22$$

$$\{4.031\} = .031$$

$$\{4\} = 0$$

$$\{-17\} = 0$$

$$\{\text{Integer}\} = 0$$

$$\{-6.22\} = 1 - .22 = .78$$

$$\{-4.013\} = 1 - .013$$

$$\{-16.29\} = 1 - .29 = .71$$

Fundamentals of Mathematics

$$\{-1.12\} = 1-.12$$

$$=.88$$

$$\{-12\} = 0$$

$$\{4.28\} = .28$$

No. of cyphers

No of Zero after Decimal.
before any significant digit
is always = $|ch| + 1$

Q No of cyphers in 2^{-100}

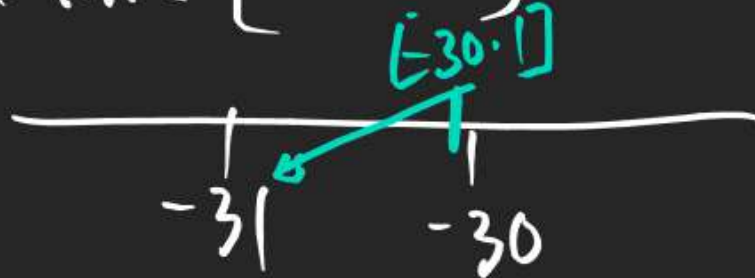
$$\log_{10} 2^{-100}$$

$$= -100 \times \log_{10} 2$$

$$= -100 \times .3010$$

$$= -30.10$$

$$(har = [-30.10] = -31$$



$$\text{No of zeros} = |-31| + 1$$

$$= |-30| = 30$$

Q No of zeros in $(\frac{2}{3})^{100}$?

$$\log_{10} (\frac{2}{3})^{100}$$

$$= 100 \times \log_{10} \frac{2}{3}$$

$$= 100 (\log_{10} 2 - \log_{10} 3)$$

$$= 100 (.3010 - .4771)$$

$$= 100 \times (-.1761)$$

$$= -17.61$$

$$ch = [-17.61] = -18$$

$$\text{No of zeros} = |-18| + 1 = 17$$

Fundamentals of Mathematics

Q Given that $\log_3 N = \alpha + \beta$ where $\beta \in [0, 1)$ ^{① Smj Jana}
K i $\beta = \text{Mantissa}$.

Now find No of Integers in N If $\alpha = 4$ ② y ha $\alpha = \text{Characteristic}$.

$$\alpha = 4 \Rightarrow ch = 4$$

$$[\log_3 N] = 4$$

$$\log_3 N \in [4, 5)$$

$$4 \leq \log_3 N < 5$$

$$3^4 \leq N < 3^5$$

$$81 \leq N < 243$$

No of Integer in N

$$= 243 - 81$$

$$= \underline{\underline{162}}$$

Fundamentals of Mathematics

Q Given that $\log_5 N = \alpha + \beta$; $\beta \in (0, 1)$

Now find the No of Integral values of N

if $\alpha = 3$.

$\alpha = 3 = (\text{characteristic})$

$$[\log_5 N] = 3$$

$$\log_5 N \in [3, 4)$$

$$3 \leq \log_5 N < 4$$

$$5^3 \leq N < 5^4$$

$$125 \leq N < 625 \Rightarrow N = 625 - 125 = 500 \text{ Int.}$$

Fundamentals of Mathematics

Modulus fcn / Absolute Value fcn.

① $f(x) = |x|$ is Modulus fcn

② $|-2| = -(-2) = 2$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|2x-1| = \begin{cases} (2x-1) & 2x-1 \geq 0 \\ -(2x-1) & 2x-1 < 0 \end{cases}$$

Aur Better.

$$|2x-1| = \begin{cases} (2x-1) & x \geq \frac{1}{2} \\ -(2x-1) & x < \frac{1}{2} \end{cases}$$

Fundamentals of Mathematics

$$|7x-4| = \begin{cases} (7x-4) & 7x-4 \geq 0 \\ -(7x-4) & 7x-4 < 0 \end{cases}$$

$$|7x-4| = \begin{cases} (7x-4) & x \geq \frac{4}{7} \\ -(7x-4) & x < \frac{4}{7} \end{cases}$$

Q Find x if $|x| = 3$
 $x = \pm 3$ ✓

$$|3| = 3$$

$$|-3| = -(-3) = 3$$

Q If $|4x-1| = 3$ then $x = ?$

$$4x-1 = \pm 3$$

$$4x-1 = 3 \quad \text{OR} \quad 4x-1 = -3$$

$$4x = 4$$

$$x = 1$$

$$4x = -3 + 1$$

$$4x = -2$$

$$x = -\frac{1}{2}$$

$$\underline{\underline{x = -\frac{1}{2}, 1}}$$

Fundamentals of Mathematics

Q $|2x+3|=5$ find x ?

$$2x+3=\pm 5$$

$$2x+3=5 \text{ or } 2x+3=-5$$

$$2x=2$$

$$x=1$$

$$x=1, -4$$

$$2x=-8$$

$$x=-4$$

Q find value of $|x-2|$ when $x < 2$

$$|x-2| = -(x-2)$$

$$= 2-x$$

$$x = 1.9 \text{ (Assume)}$$

$$|1.9-2|$$

$$|-ve|$$

Q find value of $|x+5|$ when $x \geq -5$

$$|x+5| = x+5$$

$$x = -4$$

$$|-4+5|$$

$$|+ve|$$

Fundamentals of Mathematics

$$|4+3|+|4-3|$$

Q Find value of $|4x-3|$ when $x < \frac{3}{4}$

$$\begin{array}{l|l} |4x-3| = -(4x-3) & x=0 \\ = 3-4x & |4 \times 0 - 3| \\ & |0-3| \\ & |-ve| \end{array}$$

Q Find value of $y = |x+3| + |x-3|$

when $x \geq 2$

$$y = (x+3) - (x-3)$$

$$y = \underline{6}$$

$$\begin{array}{l|l} x \geq 2 & \\ x = 2.1 & \\ [2, 3] & \end{array}$$

$$\begin{array}{l|l} |2.1+3| + |2.1-3| & \\ \sim & \\ | +ve | + | -ve | & \end{array}$$

Q Find value of $y = |x+2| + |x-2|$ when $x \geq 2$

$$y = (x+2) + (x-2)$$

$$y = 2x$$

$$x \geq 2$$

$$x = 3$$

$$\begin{array}{l|l} |3+2| + |3-2| & \\ \oplus & \oplus \end{array}$$

$$x \geq 3$$

$$x = 4$$

$$|4+3| + |4-3|$$

$$\oplus$$

$$\oplus$$

$$y = |x+3| + |x-3|$$

$$= x+x+x-x$$

$$y = \underline{2x}$$

Fundamentals of Mathematics

① $y = |3x - 4| + |5x + 1|$ has (r.pt)?

$$\begin{array}{l|l} \downarrow & 5x+1=0 \\ 3x-4=0 & \\ x=\frac{4}{3} & x=-\frac{1}{5} \end{array}$$

Q $y = |x+3| + |x-3|$ has cr pts?

Hand-drawn number line for the inequality $x < -3$ or $3 < x \leq 3$. The number line has points at -3 and 3. The region to the left of -3 is shaded green and labeled $x < -3$. The region between -3 and 3 is shaded pink and labeled $3 < x \leq 3$. The region to the right of 3 is shaded yellow and labeled $x > 3$. Arrows point down to -3 and 3, with labels -3 and 3 respectively. The number line is labeled with 3, -3, and 3.

Q $y = |x+2| + |x-3|$ has r.h.s

2 Interval?

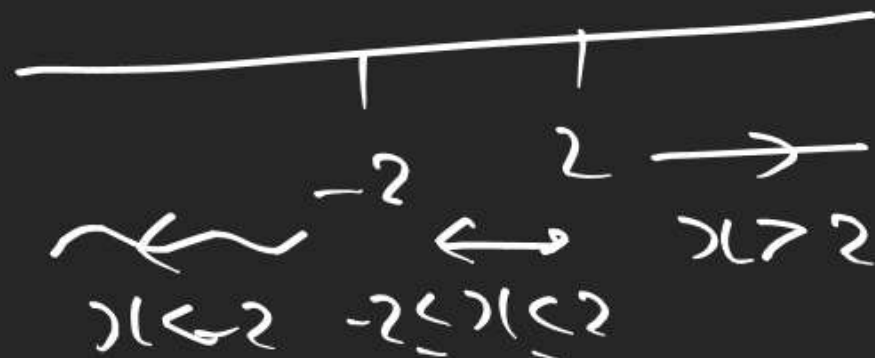
$$(r, wt) = -2, 3$$



Fundamentals of Mathematics

Q Solve $|x+2| + |x-2| = 5$

$\downarrow \quad \downarrow$
 $(x = -2 \text{ \& } 2)$



① When $x > 2 \rightarrow x = 2.1$
 $|2.1+2| + |2.1-2|$
 $\oplus \quad \oplus$
 $(x+x) + (x-x) = 5$
 $2x = 5 \Rightarrow x = \frac{5}{2} = 2.5$

② When $-2 \leq x \leq 2$

$x+2 - (x-2) = 5$
 $4 = 5$
 ϕ

$x=1$
 $|1+2| + |1-2|$
 $\oplus \quad \ominus$

$x = \frac{5}{2}, -\frac{5}{2}$

③ When $x < -2$

$x = -2.1$
 $-(-2.1) - (-2.1 - 2) = 5$
 $-2x = 5$
 $x = -\frac{5}{2} = -2.5$

$x = -2.1$
 $|-2.1+2| + |-2.1-2|$
 $\ominus \quad \ominus$

Fundamentals of Mathematics

Q $|x+3|+|x-2|=7$ Solve



① When $x < -3$

$$-(x+3)-(x-2)=7$$

$$-2x=8$$

$$x=-4 \checkmark$$

$$x=-3.1$$

$$|-3.1+3|+|-3.1-2|$$

$\ominus \quad \ominus$

② When $-3 \leq x < 2$

$$(x+3)-(x-2)=7$$

$$5=7$$

$$\otimes \quad \phi$$

$$x=-2.5$$

$$|-2.5+3|+|-2.5-2|$$

$\oplus \quad \ominus$

③ $x > 2 \checkmark$

$$(x+3)+(x-2)=7$$

$$2x=6$$

$$x=3$$

$$x=2.1$$

$$|2.1+3|+|2.1-2|$$

$\oplus \quad \oplus$

$$x=3, -4$$

Fundamentals of Mathematics

Q If $|x+1| = -4$ find $x = ?$

$$|x+1| = -4$$

$$\begin{matrix} \geq 0 & & -ve \\ \oplus & \neq & \end{matrix}$$

$$\emptyset$$

Q Find value of x satisfying $|x-4| + |x-3| = 7$

Q _____ $|x-1| + |x-3| = 4$

Q _____ $|2x-1| + |x| = 3.$